

Math 255 Practice Final

1. Find the solution of the initial value problem:

a. $y' = \frac{x^2 - 1}{y^2 + 1}, y(-1) = 1$

b. $y'' + 8y' - 9y = 0, y(1) = 1, y'(1) = 0$

c. $y''' - 3y'' + 2y' = t + e^t; y(0) = 1, y'(0) = -\frac{1}{4}, y''(0) = -\frac{3}{2}$

2. Find the solution of the differential equation:

a. $y' + y = \frac{1}{1 + e^x}$

b. $(3y^2 + 2xy)dx - (2xy + x^2)dy = 0$

c. $25y'' - 20y' + 4y = 0$

3. A tank initially contains 100 gal of pure water. Then at time $t = 0$ brine containing 0.5 lb of salt per gallon of brine is allowed to enter the tank at a rate of 2 gal/min and the mixed solution is drained from the tank at the same rate. After 10 min the process is stopped and fresh water is poured into the tank at a rate of 2 gal/min, with the mixture leaving at the same rate. How much salt is in the tank after 10 more minutes?

4. Find the general solution:

a. $9y'' + 9y' - 4y = 0$

b. $y'' + y = 3\sin 2t + t \cos 2t$

c. $y'' + y = \tan t, 0 < t < \frac{\pi}{2}$

5. Use the method of reduction of order to find a second solution of:

$$t^2 y'' + 3ty' + y = 0, t > 0; y_1(t) = t^{-1}$$

6. Find a power series solution about the given point. Find the recurrence relation and the first four terms in each of two linearly independent solutions.

$$y'' + xy' + 2y = 0, x_0 = 0$$

7. Find all singular points, and determine whether each one is regular or irregular: $xy'' + y' + (\cot x)y = 0$.

8. Find the general solution that is valid in any interval not including the singular point: $x^2y'' + 6xy' - y = 0$.

9. Find all the regular singular points of the equation. Determine the indicial equation and the exponents at the singularity for each regular singular point.

$$x^2y'' + 3(\sin x)y' - 2y = 0$$

10. Use the Laplace transform to solve the initial value problems:

a. $y'' - 4y' + 4y = 0, y(0) = 1, y'(0) = 1$

b. $y'' + 3y' + 2y = u_2(t), y(0) = 0, y'(0) = 1$

c. $y'' - y = -20d(t), y(0) = 1, y'(0) = 0$