

Math 245 Practice Final

1. Construct a truth table for  $(p \vee q) \rightarrow (p \wedge \neg r)$ .
2. Find a domain for the quantifiers that make the statement true:

$$\exists x \exists y (x \neq y \wedge \forall z ((z \neq x) \vee (z \neq y)))$$

3. Find the negation of the statement: "Everyone in this class understands mathematical induction."
4. Prove that if  $x^3$  is irrational, then  $x$  is irrational.
5. Let  $A$  and  $B$  be sets. Show that  $A \subseteq B$  if and only if  $A \cap B = A$ .
6. For which real numbers  $x$  and  $y$  is it true that  $\lceil x + y \rceil = \lceil x \rceil + \lceil y \rceil$ ?
7. True or false:  $\{x\} \in \{\{x\}\}$
8. Determine whether  $f : Z \times Z \rightarrow Z$  is onto if  $f(m, n) = m^2 - 4$ .
9. Evaluate  $\sum_{i=0}^4 (-2)^i$ .
10. Give a big-O estimate of the product of the first  $n$  odd positive integers.
11. List five integers that are congruent to 3 modulo 12.
12. Find an inverse of 4 modulo 91.
13. Find all solutions to  $x \equiv 16 \pmod{21}$ .
14. Prove using mathematical induction for nonnegative integers  $n$ :
$$1^2 + 3^2 + 5^2 + \dots + (2n+1)^2 = \frac{(n+1)(2n+1)(2n+3)}{3}$$
15. Which amounts of postage can be formed using just 4-cent stamps and 11-cent stamps? Prove your answer using strong induction.
16. Give a recursive definition of the set of odd positive integers.

17. In how many ways can a photographer at a wedding arrange 6 people in a row from a group of 10 people, where the bride and groom are among these 10 people, if the bride must be in the picture?
18. Suppose that there are nine students in a discrete mathematics class. Show that there must be at least five male students or at least five female students.
19. How many subsets with an odd number of elements does a set with 10 elements have?
20. Suppose that a club contains 10 men and 13 women. How many ways are there to form a committee with six members if it must contain more women than men?
21. What is the probability that a die never comes up an even number when it is rolled six times?
22. What is the probability that in a family with five children does not have a boy, if the sexes of children are independent and both genders are equally likely?
23. For which values of  $n$  is  $K_n$  bipartite?
24. For which integers  $n$  is  $C_n$  self-complementary?
25. Is the shortest path between two vertices in a weighted graph unique if the weights of the edges are different?
26. Suppose that a connected planar graph has six vertices, each of degree four. Into how many regions is the plane divided by a planar representation of this graph?