

### Math 245 Practice Test 3

1. Give a recursive definition of the set of positive integer powers of 3.
2. Prove that in a bit string, the string 01 occurs at most one more time than the string 10.
3. Give a recursive algorithm for finding the sum of the first  $n$  positive integers.
4. How many functions are there from the set of the first  $n$  positive integers to the set  $\{0, 1\}$ ?
5. How many numbers must be selected from the set  $\{1, 3, 5, 7, 9, 11, 13, 15\}$  to guarantee that at least one pair of these numbers add up to 16?
6. Show that in a group of five people where any two people are either friends or enemies, there are not necessarily three mutual friends or three mutual enemies.
7. Suppose that a club contains 10 men and 13 women. How many ways are there to form a committee with six members if it must contain more women than men?
8. Give a formula for the coefficient of  $x^k$  in the expansion of  $\left(x^2 - \frac{1}{x}\right)^{100}$ , where  $k$  is an integer.
9. What is the probability that a five-card poker hand contains a straight flush (five cards of the same suit of consecutive kinds)?
10. What is the probability that in a group of  $n$  people chosen at random, there are at least two born on the same day of the week?
11. In a round-robin tournament, Buster beat Jack, Buster beat Lucky, Jack beat Penelope, Jack beat Max, and Penelope beat Max. Model this outcome with a directed graph.