

Math 252 Practice Final

1. Find parametric equations of the tangent line to the curve $c(t) = (e^t \cos t, e^t \sin t, e^t)$ at $t = 0$.
2. Find an equation of the plane that contains the point $(2, 0, 3)$ and the line $x = -1 + t, y = t, z = -4 + 2t$.
3. Find the derivative matrix of $f \circ g(-1,1)$ if $f(u, v, w) = (u^2 - v^2, 3w, uvw)$ and $g(x, y) = (x, -y, xy^2)$.
4. Find the tangent plane to the graph of $f(x, y) = x^{\frac{1}{2}} + y^{\frac{1}{2}}$ at the point $(4, 9, 5)$.
5. Find and classify all the relative extrema of $f(x, y) = xe^y$.
6. Find the maximum and minimum values of $f(x, y) = x^2 - y$ subject to the constraint $x^2 + y^2 = 25$.
7. Find the arclength of $c(t) = (\sin(e^t) \cos(e^t), \sqrt{3}e^t)$ for $0 \leq t \leq 1$.
8. Find the unit tangent vector to the curve $c(t) = (e^t, e^{-t}, t)$ at $t = 0$.
9. Evaluate $\int_0^1 \int_0^{\sqrt{1-y^2}} \cos(x^2 + y^2) dx dy$.
10. Use a triple integral to find the volume of the solid bounded by the sphere $x^2 + y^2 + z^2 = 8$ and the planes $z = 0$ and $z = 2$.
11. Find the surface area of the portion of the surface $z = xy$ that is above the sector in the first quadrant bounded by the lines $y = \frac{x}{\sqrt{3}}, y = 0$, and the circle $x^2 + y^2 = 9$.
12. Evaluate $\int_C -y dx + x dy$ along $y^2 = 3x$ from the point $(3, 3)$ to the point $(0, 0)$.
13. Use Green's theorem to evaluate $\int_C x^2 y dx - y^2 x dy$ where C is the boundary of the region in the first quadrant enclosed between the coordinate axes and the circle $x^2 + y^2 = 16$.

14. Use Gauss' Divergence theorem to evaluate $\iint_S F \cdot dS$ if $F(x, y, z) = (x, y, z)$ and S is the surface of the solid bounded by the paraboloid $z = 1 - x^2 - y^2$ and the xy-plane.

15. Use Stokes' theorem to evaluate $\oint_C F \cdot dr$ if $F(x, y, z) = \langle z^2, 2x, -y^3 \rangle$ and C is the circle $x^2 + y^2 = 1$ with counterclockwise orientation looking down the positive z-axis.