

Math 121 Practice Final Exam

1. Use the definition of the derivative to find $f'(x)$ if

$$f(x) = -\frac{1}{x}.$$

2. Find the first order partial derivatives of

$$f(x, y) = x^2y + \frac{x}{y^2}.$$

3. Find $\lim_{x \rightarrow 0} f(x)$ if $f(x) = \begin{cases} x - 1 & x \leq 0 \\ -1 & x > 0 \end{cases}$

4. Find $\lim_{x \rightarrow 0} \frac{x^2 - x}{x}$.

5. Find an equation of the tangent line to

$$f(x) = \frac{3}{2x} \text{ at } \left(1, \frac{3}{2}\right).$$

6. Find $f'(x)$ if:

a. $f(x) = \sqrt{\ln x + x}$

b. $f(x) = 4e^{3x+2}$

c. $f(x) = \frac{\sqrt{x}}{x^2 + 1}$

d. $f(x) = (3x + 1)(x^2 - 2x)$

7. A farmer has 3000 yards of fencing and wishes to enclose a rectangular area along a straight river. If no fence is needed along the river, find the dimensions of the rectangle that will maximize its area.

8. Find $\lim_{x \rightarrow \infty} \frac{2x^2 + 3x + 1}{x^4 - x^2}$.

9. Evaluate:

a. $\int \left(xe^{x^2} - \frac{x}{x^2 + 2} \right) dx$

b. $\int_1^4 \frac{2x^3 - x + 5}{x^2} dx$

c. $\int_0^4 \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$

d. $\int_0^1 (x - 3)e^{3x} dx$

e. $\int_{-\infty}^0 e^{2x} dx$