

Math 151 Practice Final

1. Evaluate:

a. $\int_{-2}^2 \ln(x+3) dx$

b. $\int \sin^2 5x dx$

c. $\int \frac{dx}{x^3 + x}$

d. $\int_0^4 x^3 \sqrt{16 - x^2} dx$

e. $\int_3^4 \frac{dx}{(x-3)^2}$

2. Solve the initial value problem: $\frac{dy}{dx} + y = 2, \quad y(0) = 1.$

3. If the sequence $\left\{ \cos\left(\frac{3}{n}\right) \right\}_{n=1}^{\infty}$ converges, find its limit.

4. If $\sum_{k=1}^{\infty} \left(\frac{1}{2^k} - \frac{1}{2^{k+1}} \right)$ converges, find its limit.

5. Determine whether the series $\sum_{k=1}^{\infty} \frac{(-1)^k \ln k}{k}$ converges conditionally, converges absolutely or diverges.

6. Determine whether the following series converge or diverge:

a. $\sum_{k=1}^{\infty} \frac{k^2}{k^3 + 1}$

b.
$$\sum_{k=1}^{\infty} \left(\frac{3}{2k-1} \right)^k$$

7. Find the radius of convergence and interval of convergence for $\sum_{k=1}^{\infty} \frac{x^k}{k^2 + k}$.

8. Find the Maclaurin series for $f(x) = \cos(x^2)$.

9. Find an equation of the ellipse with foci (1, 2) and (1, 4) and major axis of length 2.

10. Graph $y^2 - 6y - 2x + 1 = 0$.

11. Express in polar coordinates: $x^2 + y^2 - 6y = 0$.

12. Find the area of the region enclosed by the rose $r = 2\cos 3\theta$.

13. Find the arc length of $x = (1+t)^3$, $y = (1+t)^2$, $0 \leq t \leq 1$.

14. Find a unit vector that is orthogonal to the vector $\mathbf{v} = (2, -1, 2)$ and to the line
$$\begin{aligned} x &= 3 - t \\ y &= 4 \\ z &= 2 + 2t. \end{aligned}$$

15. Find an equation of the line that contains the point (0, 2, 1) and intersects the line $x = 2t$, $y = 1 - t$, $z = 2 + t$ at a right angle.