Math 122 Practice Final

1. Evaluate:

   a. $\int x\cos 2x\,dx$

   b. $\int_1^4 x\ln x\,dx$

   c. $\int_{-\infty}^0 \frac{1}{x-3}\,dx$

   d. $\int_2^\infty e^{-x}\,dx$

2. Compute the first and second order partial derivatives of $f(x,y) = y\ln(x^2+\sin y)$.

3. Find and classify the relative extrema of $f(x,y) = 2x^2 + y^2 - 8x - 6y + 4$.

4. Find the maximum and minimum values of $f(x,y) = -3x^2 - y^2 + 2xy$ subject to the constraint $2x + y = 4$.

5. Find the volume of the solid bounded above by $f(x,y) = 2xe^y$ and below by the region bounded by $y = x$, $y = 2$ and $x = 0$.

6. Solve:

   a. $y' = \frac{y\ln x}{x}$

   b. $y' = x^2(1-y);\ y(0) = -2$

7. Show that $f(x) = \frac{1}{2}e^{-\frac{x}{2}}$ is a probability density function on $[0,\infty)$.

8. The function $f(x) = \frac{1}{9}x^2$ is a probability density function on [0, 3]. Find $P(x \le 2)$.

9. Find the mean, variance and standard deviation of the random variable x associated with the probability density function $f(x) = \frac{8}{7x^2}$ over [1, 8].

10. Given that $f(x, y) = xy$ is a joint probability density function on $D = \{(x, y) | 0 \leq x \leq 1, 0 \leq y \leq 2\}$ find $P(\{(x, y) | x + 2y \leq 1\})$.

11. Determine whether the sequence $a_n = \frac{1 + (-1)^n}{3^n}$ converges or diverges. If it converges, find its limit.

12. Find the sum, if it converges: $\sum_{n=1}^{\infty} \left(\frac{\pi}{e}\right)^n$.

13. Determine whether the series converges or diverges:

a. $\sum_{n=1}^{\infty} \frac{n+3}{2n^2 - 1}$

b. $\sum_{n=1}^{\infty} \frac{n}{n^3 + 4}$

14. Find the radius and interval of convergence for $\sum_{n=2}^{\infty} \frac{x^n}{n(n+1)}$.

15. Find the Taylor series for $f(x) = xe^{x^2}$ at x = 0. Give its interval of convergence.

16. Find $f'(x)$ (and do not simplify!) if:

a. $f(x) = \sec(x^2)$

b. $f(x) = \frac{\sin x}{x}$

17. Evaluate:

a. $\int (2 \sec x \tan x + \cos x) dx$

b. $\int t \sin(t^2) dt$